The optimal factoring type with partial credit guarantee in the textile industry: disclosed or undisclosed DOI: 10.35530/IT.074.02.202222

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ABSTRACT – REZUMAT

The optimal factoring type with partial credit guarantee in the textile industry: disclosed or undisclosed

Small and medium-sized textile enterprises generally experience financial difficulties, with a high default risk because of the textile industry's characteristics of low concentration, long industrial chains, and large seasonal fluctuations. Using a two-echelon supply chain model of the textile industry comprising a core retailer and capital-constrained supplier, this study investigates disclosed and undisclosed factoring while considering the default risk of both the supplier and retailer under two guarantee mechanisms: no guarantee and a third-party partial credit guarantee. Utilizing a Stackelberg game model, this study finds that both the default risk and financial institutions' loan-to-value ratio for accounts receivable significantly affect optimal financing decisions and financing efficiency. First, excessively pursuing higher loan-to-value ratios lowers financing efficiency. In addition, a partial credit guarantee from a third party can effectively reduce the financing interest rate but cannot improve financing efficiency if the supplier assumes the guarantee fee. Thus, while introducing a guarantee mechanism to control financing risk, financial institutions should consider supply chain participants, rather than the supplier, to assume the guarantee fees. Furthermore, both the supplier and retailer should finance through disclosed factoring regardless of a guarantee. Our findings offer textile industry-specific financing insights regarding the options of guarantee and factoring financing type based on the accounts receivable.

Keywords: factoring, guarantee, game model, default risk, efficiency

Tipul optim de factoring cu garanție parțială a creditului în industria textilă: divulgat sau nedivulgat

Întreprinderile textile mici și mijlocii se confruntă în general cu dificultăți financiare, cu un risc mare de nerambursare din cauza caracteristicilor industriei textile de concentrare scăzută, lanțuri industriale lungi și fluctuații sezoniere mari. Folosind un model de lanț de aprovizionare cu două eșaloane al industriei textile, care cuprinde un comerciant cu amănuntul de bază și un furnizor cu capital limitat, acest studiu investighează factoringul divulgat și nedivulgat, luând în considerare riscul de neplată atât al furnizorului, cât și al comerciantului cu amănuntul în cadrul a două mecanisme de garantare: fără garanție și garanția parțială a creditului pentru terța parte. Folosind un model de joc Stackelberg, acest studiu constată că atât riscul de nerambursare, cât și raportul împrumut-valoare al instituției financiare pentru conturile de încasat afectează în mod semnificativ deciziile optime de finanțare și eficiența finanțării. În primul rând, urmărirea excesivă a unor rapoarte credit/valoare mai ridicate scade eficiența finanțării. În plus, o garanție parțială a creditului de la o terță parte poate reduce efectiv rata dobânzii de finanțare, dar nu poate îmbunătăți eficiența finanțării dacă furnizorul își asumă comisionul de garanție. Astfel, în timp ce se introduce un mecanism de garantare pentru controlul riscului de finanțare, instituțiile financiare ar trebui să ia în considerare participanții lanțului de aprovizionare, mai degrabă decât furnizorul, să își asume taxele de garanție. În plus, atât furnizorul, cât și comerciantul cu amănuntul ar trebui să finanțare prin factoring divulgat, indiferent de garanție. Constatările noastre oferă perspective de finanțare specifice industriei textile cu privire la opțiunile de finanțare cu garanție și factoring pe baza conturilor de încasat.

Cuvinte-cheie: factoring, model de joc, risc implicit, eficiență

INTRODUCTION

The textile industry is an important pillar of people's livelihood and employment generation. However, because the textile industry has a long industrial chain and many subdivisions, approximately 80% of textile enterprises are small- and medium-sized enterprises (SMEs). These SMEs generally experience financing difficulties and a high default risk because of problems such as information asymmetry and the textile industry's characteristics of low concentration, long industrial chains, and large seasonal fluctuations.

The capital constraint is a key factor that commonly affects firms' operations, especially those of SMEs. In

particular, the COVID-19 outbreak has severely affected manufacturing industries such as textiles; for example, many enterprises – even large or core ones – have suffered from raw material and capital shortages. SMEs face a more severe situation because of the shortage of cash flows, especially those with accounts receivable as their main business in supply chain operations, with longer account periods and greater financing difficulties.

A potential method to free up cash flows is factoring in financing. Factoring is a type of short-term supplier financing in which suppliers sell their accounts receivable receipts at a discount (loan-to-value ratio) and receive immediate cash from factors such as banks or other financial institutions [1]. Factoring can be disclosed or undisclosed based on whether the core enterprise is notified. Disclosed factoring (DF) seems to be the main form of factoring. However, with higher pressures on capital flow and supplier selection, an increasing number of capital-constrained enterprises, especially suppliers, have begun utilizing undisclosed factoring (UF). For example, as the largest brokerage company specializing in insurance and financing of receivables and a partner of the A.U. Group, ARFIN secures business transactions worth more than EUR 200 billion and has over 15 years of experience in the field of factoring, including recourse factoring, reverse factoring, and UF. However, UF is subject to two risks, as the core enterprise (the debtor) is not notified about the transfer of accounts receivable by the capital-constrained supplier (the creditor): i) The core enterprise may default, and ii) the capital-constrained supplier may cheat on loans through forged receipts. This reveals that the factoring company must conduct strict reviews before granting credit to the capital-constrained suppliers to mitigate and prevent financing risks from UF.

In this financing environment, financial institutions, such as banks, usually require a guarantee mechanism in the process of factoring to reduce financing risks. Many studies have examined the guarantee issues in financing from various perspectives, such as third-party [2–4], insurance [5], and core enterprise [6]. However, to the best of our knowledge, few have considered the guarantee issue under UF. Furthermore, no study has performed a comparative analysis of the optimal decisions and financing efficiency between DF and UF under guarantee to provide insights into the optimal factoring type in the context of the textile industry.

Here, we aim to answer the following research questions:

- How does a guarantee affect the financing decisions under both DF and UF?
- In the textile industry, what is the optimal factoring type between DF and UF under both guarantee and no guarantee scenarios?

Using a two-echelon supply chain comprising a core retailer and capital-constrained supplier, this study analyses the impact of guarantee on the optimal factoring decisions and types while considering the default risks of the retailer and supplier utilizing a Stackelberg game model. We show that the bank's loan-to-value ratio for accounts receivable and default risk significantly affect financing decisions and profits. First, the profits of both the supplier and retailer decrease with the loan-to-value ratio and default risk. Thus, the capital-constrained supplier cannot always benefit from a higher loan-to-value ratio because of higher financing costs. In addition, a partial credit guarantee (PCG) from a third party can effectively reduce the financing interest rate but cannot improve the financing efficiency if the supplier assumes the guarantee fee. Furthermore, the optimal profits of both the supplier and retailer under DF are always higher than those under UF regardless of a guarantee.

This study makes several contributions. First, from a theoretical perspective, our study is the first to analyse the optimal factoring type between DF and UF in the textile industry while considering guarantee. Studies on factoring mainly focus on DF [1,7] and reverse factoring [1,8]. Meanwhile, studies in the field of guarantee mainly focus on scenarios such as PCG [9, 10], buy-back [3, 11], and core enterprise guarantee [6, 12]; rarely do they comprehensively consider the guarantee under both DF and UF, especially the optimal factoring type between the two. Finally, studies on the textile industry primarily focus on material reserves [13, 14], inventory management [15], and supply chain management [16, 17]. Our study fills the research gap on factoring and guarantee under UF in the textile industry.

Second, by using the Stackelberg game model, our study provides some guidelines for supply chain participants and financial institutions on the factoring business in the textile industry, especially under UF. We show that a capital-constrained supplier may not find it profitable to pursue a higher loan-to-value ratio for accounts receivable. In addition, while introducing a guarantee mechanism to control financing risk, banks should consider supply chain participants, rather than the supplier, to assume the guarantee fees. Finally, both the supplier and retailer should finance through DF regardless of a guarantee.

The remainder of this paper is organized as follows. Section 2 discusses DF and UF models under different guarantee scenarios. Section 3 explores the impact of guarantee and factoring types on financing efficiency. Finally, Section 4 presents the conclusions of this study.

DISCLOSED AND UNDISCLOSED FACTORING MODELS

Model setup

We consider a two-echelon supply chain comprising a capital-constrained supplier and a core retailer. After the supplier offers the product to the retailer, its capital is not enough to support its operations, and it needs financing from banks and other financial institutions for accounts receivable. The supplier can choose between two types of factoring: DF and UF. Under DF, the supplier enters into a factoring agreement with the bank and assigns the benefit of the debts created by the sales transaction to them. The retailer is then notified and sends payment to the bank. The DF arrangement is usually on a nonrecourse basis. Meanwhile, UF, which is usually undertaken on a recourse basis, does not involve the retailer. The agreement is made between the bank and supplier, and the retailer must pay as per the sales contract. While receiving payments, the supplier holds the funds in a separate bank account as a trustee for the bank. Finally, we also consider two scenarios: no guarantee and PCG.







Fig. 2. The sequence of events in DF and UF with PCG from the third-party

Scenario 1: No guarantee. The sequence of events in this scenario is shown in figure 1.

As shown in figure 1, at the end of the sales period, the sequence of clearing accounts under DF is as follows: The bank pays the remaining credit to the supplier if the retailer pays the bank (and does not default), and the supplier pays the financing interest to the bank simultaneously. Otherwise, if the retailer defaults, the bank claims financing interest from the supplier and does not pay the remaining credit. Meanwhile, the sequence of clearing accounts at the end of the sales period under UF is as follows: If the retailer does not default, it pays the supplier first. The supplier then pays the financing amount and interest to the bank, which then pays the remaining credit to the supplier. If the retailer defaults at the end of the sales period, the bank seeks direct recourse with the supplier and does not pay the remaining credit. **Scenario 2:** PCG from the third party. The sequence of events is shown in figure 2.

	Table 1
NOTATIONS	
Parameters	Decision variables
<i>p</i> : Retailer's unit retail price in the market	w _{ij} : Wholesale price
<i>c</i> : Supplier's unit production cost	<i>q_{ij}</i> : Order quantity
α : The probability of retailer default	<i>r_{ij}</i> : Bank's financing interest under
β : The probability of supplier default	Functions:
λ : The PCG coefficient	Π_{ij}^{t} : The expected profit
δ : The unit guarantee fee for the third-party PCG	$q(p,\varepsilon)$: Demand curve
r_{f} : The risk-free interest rate	Abbreviations
ϵ : Random variable with mean 0, variance σ^2	R: Retailer; S: Supplier; SC: Supply Chain
θ_{ij} : Bank's loan-to-value ratio to the supplier under scenario I and factoring type <i>j</i>	D&U: Disclosed & Undisclosed factoring
	<i>i</i> = 1,2: no guarantee, third-party offers PCG
A: Supplier's asset (asset can be recourse)	<i>j</i> = D,U; <i>t</i> = R,S,SC



As shown in figure 2, the sequence of events under both DF and UF is similar to figure 1. The only difference in the PCG scenario is that the bank can claim a share (guarantee coefficient) of the loan loss from the third party when the retailer defaults.

Notations in this paper are summarized in table 1. Our assumptions are as follows:

- 1. The bank, retailer, supplier, and the third party are all risk neutral. The bank's market is competitive.
- 2. The default risk of both the retailer and supplier is exogenous, which may be driven by the exogenous credit shock associated with the credit rating or moral hazard [1, 18].
- 3. Similar to the demand curve used in [19] and [20], we assume that $q(p,\varepsilon) = a bp + \varepsilon$, where ε has mean 0 and variance σ^2 .
- 4. The salvage value of unsold products at the end of the sales period is zero. Furthermore, we assume that $\lambda < \theta_{ij} < 1$; that is, the PCG coefficient does not exceed the bank's loan-to-value ratio for the accounts receivable.

DF model under no guarantee

The sequence of events is shown in figure 1. First, the supplier decides w_{1D} based on equation 1.

$$\Pi_{1D}^{S}(w_{1D}) = \max_{w_{1D}} \theta_{1D} w_{1D} q_{1D} + (1-\alpha)(1-\theta_{1D}) w_{1D} q_{1D} - w_{1D} q_{1D} r_{1D} - cq_{1D}$$
(1)

The first term on the right side of equation 1 is the bank's financing amount for accounts receivable; the second is the remaining financing amount that the bank should pay if the retailer does not default; and the third and fourth terms are the supplier's financing interest and production, respectively.

The retailer's decision on its order quantity, q_{1D} , can then be formulated as equation 2.

$$\Pi_{1D}^{R}(q_{1D}) = \max_{q_{1D}} E_{\varepsilon}[pq(p,\varepsilon)] - w_{1D}q_{1D}(1-\alpha) \quad (2)$$

The first term on the right side of equation 2 is the retailer's sales revenue. The second term is the payment to the bank if the retailer does not default.

Finally, the bank decides the financing rate, r_{1D} , which can be formulated as equation 3.

$$\theta_{1D} w_{1D} q_{1D} (1+r_f) = (1-\alpha) \theta_{1D} w_{1D} q_{1D} + w_{1D} q_{1D} r_{1D}$$
(3)

The left side of equation 3 is the bank's risk-free income. The first term on the right side is the payment received by the bank if the retailer does not default and the second is the financing interest.

Equations 1–3 represent a Stackelberg game. We proceed backwards and derive Lemma 1.

Lemma 1: When no PCG in DF,
$$w_{1D}^* = \frac{a\rho_2 + bc\rho_1}{2b\rho_1\rho_2}$$
,
 $q_{1D}^* = \frac{a\rho_2 - bc\rho_1}{4\rho_2}$, $r_{1D}^* = \theta_{1D}(\alpha + r_f)$, where $\rho_1 = 1 - \alpha$,
 $\rho_2 = \rho_1 - \theta_{1D}r_f$.

Proposition 1: In the DF model when no PCG,

(1)
$$w_{1D}^* \propto \alpha$$
, $q_{1D}^* \propto \frac{1}{\alpha}$, $r_{1D}^* \propto \alpha$, and
 $\Pi_{1D}^S = \frac{(a\rho_2 - bc\rho_1)^2}{8b\rho_1\rho_2} \propto \frac{1}{\alpha}$, $\Pi_{1D}^R = \frac{(a\rho_2 - bc\rho_1)^2}{16b(\rho_2)^2} \propto \frac{1}{\alpha}$

where " \propto " means a positive effect. For example, $w_{1D}^* \propto \alpha$ means increases with α ;

(2)
$$w_{1D}^* \propto \theta_{1D}, \quad q_{1D}^* \propto \frac{1}{\theta_{1D}}, \quad r_{1D}^* \propto \theta_{1D}, \text{ and}$$

$$\Pi_{1D}^S \propto \frac{1}{\theta_{1D}}, \quad \Pi_{1D}^R \propto \frac{1}{\theta_{1D}}.$$

In factoring practice, the supplier usually expects the bank to provide a larger payment (loan-to-value ratio) for the accounts receivable, thereby reducing their own risk when a retailer defaults. However, Proposition 1 shows that a higher loan-to-value ratio will lead to lower financing efficiency. This is because the bank will set a higher financing interest rate to avoid financing risks while providing a larger loan-tovalue ratio because of the retailer's default risk. Consequently, the supplier experiences higher financing costs. This increases the wholesale price and lowers the order quantity, consequently lowering the supply chain's financing efficiency.

UF model under no guarantee

The sequence of events is shown in figure 1. Similarly, the supplier decides w_{1U} based on equation 4.

$$\Pi_{1U}^{S}(w_{1U}) = \max_{w_{1U}} \theta_{1U} w_{1U} q_{1U} + (1-\alpha)(1-\theta_{1U}) w_{1U} q_{1U} - w_{1U} q_{1U} r_{1U} - cq_{1U} - \alpha A$$
(4)

The fifth term is the amount of the bank's recourse to the supplier if the retailer defaults.

Then the retailer decides on the orders based on equation 5.

$$\Pi_{1U}^{R}(q_{1U}) = \max_{q_{1U}} E_{\varepsilon}[pq(p,\varepsilon)] - w_{1U}q_{1U}(1-\alpha) \quad (5)$$

Finally, the bank decides the financing rate r_{1U} , which can be formulated as equation 6.

$$\theta_{1U} w_{1U} q_{1U} (1+r_f) = (1-\alpha) [(1-\beta) \theta_{1U} w_{1U} q_{1U} + \beta A] + + \alpha A + w_{1U} q_{1U} r_{1U}$$
(6)

The first term on the right side of equation (6) is the bank's expected revenue received from the supplier if the retailer does not default; the second is the payment received by the bank from the supplier if the retailer defaults; and the third term is the financing interest.

Similarly, we proceed backwards and derive Lemma 2 based on equations 4 to 6.

Lemma 2: When no PCG in UF, and without loss of generality, we assume that the supplier's asset can be recourse is zero (*A*=0), then $w_{111}^* = \frac{a\rho_3 + bc\rho_1}{Ct}$.

$$q_{1U}^* = \frac{a\rho_3 - bc\rho_1}{4\rho_3}, r_{1U}^* = \theta_{1U}[(1 - \alpha)\beta + \alpha + r_f], \text{ where}$$

 $\rho_3 = (1\!-\!\alpha)(1\!-\!\beta\theta_{1U}) - \theta_{1U}r_f.$

Lemma 2 indicates that both the retailer's default and supplier's default have a large impact on the financing decisions and financing efficiency under UF, which is shown in Proposition 2 below.

Proposition 2: In the UF model when no PCG,

(1)
$$w_{1U}^* \propto \alpha$$
, $q_{1U}^* \propto \frac{1}{\alpha}$, $r_{1U}^* \propto \alpha$, and

$$\Pi_{1U}^S = \frac{(a\rho_3 - bc\rho_1)^2}{8b\rho_1\rho_3} \propto \frac{1}{\alpha}$$
, $\Pi_{1U}^R = \frac{(a\rho_3 - bc\rho_1)^2}{16b(\rho_3)^2} \propto \frac{1}{\alpha}$
(2) $w_{1U}^* \propto \beta$, $q_{1U}^* \propto \frac{1}{\beta}$, $r_{1U}^* \propto \beta$, and

$$\Pi_{1U}^{S} \propto \frac{1}{\beta}, \ \Pi_{1U}^{R} \propto \frac{1}{\beta};$$
(3) $w_{1U}^{*} \propto \theta_{1U}, \ q_{1U}^{*} \propto \frac{1}{\theta_{1U}}, \ r_{1U}^{*} \propto \theta_{1U}, \text{ and}$

$$\Pi_{1U}^{S} \propto \frac{1}{\theta_{1U}}, \ \Pi_{1U}^{R} \propto \frac{1}{\theta_{1U}}.$$

Similar to Proposition 1, under UF, both the supplier's wholesale price and the bank's financing interest rate increase with the retailer's default risk and the bank's loan-to-value ratio; meanwhile, the retailer's orders and the profits of both the supplier and retailer decrease. Therefore, under UF, excessively pursuing higher loan-to-value ratios for the accounts receivable increases (decreases) financing costs (financing efficiency). In addition, as the supplier's default risk increases under UF, the bank sets a higher interest rate, which increases financing costs for the capital-constrained supplier, consequently lowering the supply chain's financing efficiency.

DF model with a third-party PCG

The sequence of events is shown in figure 2. Similarly, the supplier decides w_{2D} based on equation 7.

$$\Pi_{2D}^{S}(w_{2D}) = \max_{w_{2D}} \theta_{2D} w_{2D} q_{2D} + (1-\alpha)(1-\theta_{2D})w_{2D}q_{2D} - w_{2D}q_{2D}r_{2D} - cq_{2D} - \delta w_{2D}q_{2D}$$
(7)

The fifth term of equation 7 represents the guarantee fees that the supplier pays to the third party.

The retailer's decision on its order quantity q_{2D} can be formulated as equation 8.

$$\Pi_{2D}^{R}(q_{2D}) = \max_{q_{2D}} E_{\varepsilon}[pq(p,\varepsilon)] - w_{2D}q_{2D}(1-\alpha) \quad (8)$$

The third party decides the guarantee coefficient λ_{2D} , which can be formulated as equation 9.

$$\lambda_{2D} w_{2D} q_{2D} \alpha = \delta w_{2D} q_{2D} \tag{9}$$

The term on the left side of equation 9 represents the guarantee costs of the third party if the retailer defaults, and the term on the right side is the third party's income by offering a PCG.

Finally, the bank decides the financing rate r_{2D} based on equation 10.

$$\theta_{2D} w_{2D} q_{2D} (1+r_f) = (1-\alpha) \theta_{2D} w_{2D} q_{2D} + + w_{2D} q_{2D} r_{2D} + \lambda_{2D} w_{2D} q_{2D} \alpha$$
(10)

Similarly, we proceed backwards and derive Lemma 3 below.

Lemma 3: When no PCG in DF,
$$w_{2D}^* = \frac{a\rho_2 + bc\rho_1}{2b\rho_1\rho_2}$$

 $q_{2D}^* = \frac{a\rho_2 - bc\rho_1}{4\rho_2}$, $r_{2D}^* = \theta_{2D}(\alpha + r_f) - \alpha\lambda_{2D}$.

Proposition 3: In DF model when third-party offers PCG,

(1)
$$w_{2D}^* \propto \alpha$$
, $q_{2D}^* \propto \frac{1}{\alpha}$, $r_{2D}^* \propto \alpha$, and

$$\Pi_{2D}^S = \frac{(a\rho_2 - bc\rho_1)^2}{8b\rho_1\rho_2} \propto \frac{1}{\alpha}$$
, $\Pi_{2D}^R = \frac{(a\rho_2 - bc\rho_1)^2}{16b(\rho_2)^2} \propto \frac{1}{\alpha}$;
(2) $w_{2D}^* \propto \theta_{2D}$, $q_{2D}^* \propto \frac{1}{\theta_{2D}}$, $r_{2D}^* \propto \theta_{2D}$, and

$$\Pi_{2D}^S \propto \frac{1}{\theta_{2D}}$$
, $\Pi_{2D}^R \propto \frac{1}{\theta_{2D}}$.

(3) For a given α and θ_{2D} , then $w_{2D}^* \perp \lambda_{2D}$, $q_{2D}^* \perp \lambda_{2D'}$, $r_{2D}^* \propto \frac{1}{\lambda_{2D}}$, $\Pi_{2D}^S \perp \lambda_{2D}$, $\Pi_{2D}^R \perp \lambda_{2D}$, where " \perp " means no relationship. For a given and interest rate r_{2D}^* , then

$$\theta_{2D} \propto \lambda_{2D}, \ w_{2D}^* \propto \lambda_{2D}, \ q_{2D}^* \propto \frac{1}{\lambda_{2D}}$$

Proposition 3 notes that the PCG coefficient significantly affects decisions and profits. First, if the bank's loan-to-value ratio remains unchanged, a third-party PCG can reduce the financing rate. Interestingly, a PCG does not affect the optimal decisions of the supplier and retailer even when the supplier assumes the guarantee fees. This is because the decline in the financing interest rate reduces the supplier's financing cost; this enables the supplier (retailer) to maintain the optimal wholesale price (optimal order quantity). In addition, if the bank sets a fixed interest rate, a third-party PCG can increase the bank's loan-tovalue ratio for the accounts receivable. However, the supplier will charge a higher wholesale price when a third party offers a PCG and the wholesale price increases with the PCG coefficient. This is because a higher PCG coefficient can reduce the bank's financing interest rate or improve its loan-to-value ratio. which in turn offsets the guarantee costs borne by the supplier.

UF model with a third-party PCG

The sequence of events is shown in figure 2. Similarly, the supplier decides w_{2U} based on equation 11.

$$\Pi_{2U}^{S}(w_{2U}) = \max_{w_{2U}} \theta_{2U} w_{2U} q_{2U} + (1-\alpha)(1-\theta_{2U}) w_{2U} q_{2U} - \alpha A - w_{2U} q_{2U} r_{2U} - -cq_{2U} - \delta w_{2U} q_{2U}$$
(11)

Then the retailer decides on the orders q_{2U} based on equation 12.

$$\Pi_{2U}^{R}(q_{2U}) = \max_{q_{2U}} E_{\varepsilon}[pq(p,\varepsilon)] - w_{2U}q_{2U}(1-\alpha)$$
(12)

The third party decides the guarantee coefficient λ_{2U} based on equation 13.

$$\lambda_{2U} w_{2U} q_{2U} [\alpha + (1 - \alpha)\beta] = \delta w_{2U} q_{2U}$$
(13)

Finally, the bank decides the financing interest rate r_{2U} based on equation 14.

$$\theta_{2U} w_{2U} q_{2U} (1+r_f) = (1-\alpha) [(1-\beta) \theta_{2U} w_{2U} q_{2U} + \beta A] + \alpha A + w_{2U} q_{2U} r_{2U} + \lambda_{2U} w_{2U} q_{2U} [\alpha + (1-\alpha)\beta]$$
(14)

Similarly, we proceed backwards and derive Lemma 4 below.

Lemma 4: When the third-party offers PCG in UF,

$$w_{2U}^{*} = \frac{a\rho_{3} + bc\rho_{1}}{2b\rho_{1}\rho_{3}}, \quad q_{2U}^{*} = \frac{a\rho_{3} - bc\rho_{1}}{4\rho_{3}},$$

$$r_{2U}^{*} = (\theta_{2U} - \lambda_{2U})[(1 - \alpha)\beta + \alpha] + \theta_{2U}r_{f}.$$

Proposition 4: In UF model when third-party offers PCG,

(1)
$$W_{2U}^* \propto \alpha$$
, $q_{2U}^* \propto \frac{1}{\alpha}$, $r_{2U}^* \propto \alpha$, and

$$\Pi_{2U}^S = \frac{(a\rho_3 - bc\rho_1)^2}{8b\rho_1\rho_3} \propto \frac{1}{\alpha}$$
, $\Pi_{2U}^R = \frac{(a\rho_3 - bc\rho_1)^2}{16b(\rho_3)^2} \propto \frac{1}{\alpha}$
(2) $W_{2U}^* \propto \beta$, $q_{2U}^* \propto \frac{1}{\beta}$, $r_{2U}^* \propto \beta$, and
 $\Pi_{2U}^S \propto \frac{1}{\beta}$, $\Pi_{2U}^R \propto \frac{1}{\beta}$;
(3) $W_{4U}^* \propto \theta_{4U}$, $q_{4U}^* \propto \frac{1}{\alpha}$, $r_{4U}^* \propto \theta_{4U}$, and

(3)
$$w_{2U}^* \propto \theta_{2U}, \quad q_{2U}^* \propto \frac{1}{\theta_{2U}}, \quad r_{2U}^* \propto \theta_{2U},$$

$$\Pi_{2U}^S \propto \frac{1}{\theta_{2U}}, \quad \Pi_{2U}^R \propto \frac{1}{\theta_{2U}}.$$

(4) For a given α , β and θ_{2U} , then $w_{2U}^* \perp \lambda_{2U'} w_{2U}^* \perp \lambda_{2U}$ $r_{2U}^* \propto \frac{1}{\lambda_{2U}}$, $\Pi_{2U}^S \perp \lambda_{2U}$, $\Pi_{2U}^R \perp \lambda_{2U}$. For a given α , β and interest rate r_{2U}^* , then $\theta_{2U} \propto \lambda_{2U}$, $w_{2U}^* \propto \lambda_{2U}$, $q_{2U}^* \propto \frac{1}{\lambda_{2U}}$.

Proposition 4 states that the impact of the default risks of both the retailer and supplier and the bank's loan-to-value ratio on the optimal decisions and profits are consistent with Proposition 2 in Section 2.3. Furthermore, the PCG coefficient significantly affects decisions and profits. First, if the bank's loan-to-value ratio remains unchanged, a third-party PCG can reduce the negative effect of default risk on the bank. The supplier's financing cost is thus lower, enabling the supplier and retailer to maintain the optimal wholesale price and order quantity. Furthermore, this PCG can increase the bank's loan-to-value ratio if the bank sets a fixed financing interest rate. However, the supplier's wholesale price increases with the PCG coefficient. This is because the decrease in the financing interest rate or increase in the loan-to-value ratio can offset the guarantee costs borne by the supplier.

IMPACT OF GUARANTEE AND FACTORING TYPE

The impact of PCG under DF and UF

For the impact of guarantee under DF, we deduce Proposition 5 based on Lemmas 1 and 3.

Proposition 5: Under DF, if
$$\theta_{1D} = \theta_{2D} = \theta$$
, then $w_{1D}^* = w_{2D}^*$, $q_{1D}^* = q_{2D}^*$, $r_{1D}^* > r_{2D}^*$, $\Pi_{1D}^S = \Pi_{2D}^S$, and $\Pi_{1D}^R = \Pi_{2D}^R$.

For the impact of guarantee under UF, we deduce Proposition 6 based on Lemmas 2 and 4.

Proposition 6: Under UF, if $\theta_{1U} = \theta_{2U} = \theta$, then $w_{1U}^* = w_{2U}^*$, $q_{1U}^* = q_{2U}^*$, $r_{1U}^* > r_{2U}^*$, $\Pi_{1U}^S = \Pi_{2U}^S$, and $\Pi_{1U}^R = \Pi_{2U}^R$.

Propositions 5 and 6 states that whether under DF or UF, a third-party PCG can effectively reduce the bank's interest rate; however, the wholesale price, order quantity, and financing efficiency remain the same as that under no guarantee. This is because the positive impact of the decrease in the interest rate or increase in the loan-to-value ratio under PCG is offset by the guarantee costs borne by the supplier. Consequently, the supplier maintains the wholesale price rather than reducing it. Therefore, the optimal profit of each participant remains unchanged. Propositions 5 and 6 show that when the supplier assumes the guarantee fee, a third-party PCG can reduce the bank's financing risk and interest rate; however, it cannot improve the supply chain's financing efficiency.

The optimal factoring type between DF and UF

For the optimal factoring type under no guarantee and a third-party PCG, we deduce Proposition 7 based on Lemmas 1 and 2 and Proposition 8 based on Lemmas 3 and 4.

Proposition 7: Under factoring with no guarantee, if $\theta_{1D} = \theta_{1U} = \theta$, then $w_{1D}^* \leq w_{1U}^*$, $q_{1D}^* \geq q_{1U}^*$, $r_{1D}^* \leq r_{1U}^*$, $\Pi_{1D}^S \geq \Pi_{1U}^S$, $\Pi_{1D}^R \geq \Pi_{1U}^R$, and $\Pi_{1D}^{SC} \geq \Pi_{1U}^{SC}$; the equality holds if and only if $\beta = 0$.

Proposition 8: Under factoring with a third-party PCG, if $\theta_{2D} = \theta_{2U} = \theta$, then $w_{2D}^* \le w_{2U}^*$, $q_{2D}^* \ge q_{2U}^*$, $r_{2D}^* \le r_{2U}^*$, $\Pi_{2D}^S \ge \Pi_{2U}^S$, $\Pi_{2D}^R \ge \Pi_{2U}^R$, and $\Pi_{2D}^{SC} \ge \Pi_{2U}^{SC}$; the equality holds if and only if $\beta = 0$.

Propositions 7 and 8 show that the financing efficiency under UF is always lower than that under DF with or without PCG. The reasons are as follows. First, the bank's financing interest rate under UF is higher than that under DF because the supplier's factoring financing does not inform the retailer (core enterprise as the debtor). Simultaneously, there is an information asymmetry between the bank and the supplier on whether the supplier will default. The bank thus assumes the double default risk from both the retailer and supplier and chooses a higher financing interest rate under UF. Therefore, the financing cost of the capital-constrained supplier increases, leading to a

higher wholesale price and lower order quantity, consequently lowering the supply chain's financing efficiency.

CONCLUSIONS

As an important pillar of people's livelihood, SMEs generally experience financing difficulties because of the textile industry's characteristics of low concentration, long industrial chains, and large seasonal fluctuations. To solve financing difficulties, this study investigates the optimal factoring type between DF and UF and the impact of guarantee on factoring. We find that an excessive pursuit of a loan-to-value ratio for the accounts receivable leads to lower financing efficiency. Furthermore, a third-party PCG can effectively reduce the financing interest rate but cannot improve the financing efficiency if the supplier assumes the guarantee fee. Finally, the profits of both the supplier and retailer are higher under DF than under UF. Our analysis provides guidelines for supply chain and financial institutions regarding the factoring business in the textile industry. First, a capital-constrained supplier may not find it profitable to pursue a higher loanto-value ratio for accounts receivable. Moreover, while introducing a guarantee mechanism to control financing risk, financial institutions should consider supply chain participants, rather than the supplier, to assume the guarantee fees. Furthermore, both the supplier and retailer should choose financing under DF regardless of a guarantee.

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